String and brane models with spontaneously or dynamically induced tension

E. I. Guendelman* and A. Kaganovich†
Department of Physics, Ben-Gurion University of the Negev, P. O. Box 653, IL-84105 Beer-Sheva, Israel

E. Nissimov‡ and S. Pacheva§
Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Boul. Tsarigradsko Chausee 72, BG-1784 Sofia, Bulgaria

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We study in some detail the properties of a previously proposed new class of string and brane models whose world-sheet (world-volume) actions are built with a modified reparametrization-invariant measure of integration and which do not contain any *ad hoc* dimensionful parameters. The ratio of the new and the standard Riemannian integration measure densities plays the role of a dynamically generated string or brane tension. The latter is identified as (the magnitude of) an effective (non-Abelian) electric field strength on the world-sheet or world-volume obeying the standard Gauss-law constraint. As a result a simple classical mechanism for confinement via modified-measure "color" strings is proposed where the colorlessness of the "hadrons" is an automatic consequence of the new string dynamics.

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I. INTRODUCTION: MAIN IDEAS AND FEATURES OF THE THEORY

One of the characteristic features of string and brane theories [1] is the introduction *ad hoc* from the very beginning of a dimensionful scale—the so called string (brane) tension. On the other hand, a lot of attention has been given to the idea that any fundamental theory of nature should not contain any *ad hoc* fundamental scales and that these scales should rather appear as a result of dynamical generation, e.g., through boundary conditions on the classical level, and spontaneous symmetry breaking and/or dimensional transmutation on the quantum level (see, for instance, Ref. [2] about spontaneous generation of Newton's gravitational constant).

In the context of string and brane theories, the above idea was first explored in Refs. [3]. In this section we will briefly review, with some additional new accents, the main properties of the modified string and brane theories of [3] in order to prepare the ground for revealing new interesting structures inherent in these theories. To this end let us first recall the standard Polyakov-type action for the bosonic string, which reads [4]

$$S_{\rm Pol}=-T\int d^2\sigma\frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}(X). \qquad (1)$$

Here $(\sigma^0,\sigma^1)\equiv(\tau,\sigma)$; $a,b=0,1$; $\mu,\nu=0,1,\ldots,D-1$; $G_{\mu\nu}$ denotes the external space-time metric; γ_{ab} is the metric defined on the $(1+1)$-dimensional world-sheet of the string; and $\gamma=\det\|\gamma_{ab}\|$. T indicates the string tension—a dimensionful quantity introduced *ad hoc* into the theory which defines a scale.

Now following Refs. [3], instead of the standard measure of integration $d^2\sigma\sqrt{-\gamma}$, we want to consider a new reparametrization-invariant measure on the string world-sheet whose density Φ is independent of the Riemannian metric γ_{ab}. This approach of considering an alternative integration measure has been studied in the context of $D=4$ gravitational theory, in particular, in relation to the cosmological constant problem [5] (and references therein), as well as the fermion families and long-range force problems [6].

Indeed, if we introduce two auxiliary scalar fields [scalars both from the point of view of the $(1+1)$-dimensional world-sheet of the string, as well as from the point of view of the embedding D-dimensional universe] φ^i $(i=1,2)$, we can construct the following world-sheet measure density:

$$\Phi(\varphi)\equiv\frac{1}{2}\varepsilon_{ij}\varepsilon^{ab}\partial_a\varphi^i\partial_b\varphi^j=\varepsilon_{ij}\dot{\varphi}^i\partial_\sigma\varphi^j. \qquad (2)$$

It is interesting to notice that $d^2\sigma\,\Phi(\varphi)=d\varphi^1 d\varphi^2$, that is, the measure of integration $d^2\sigma\,\Phi$ corresponds to integrating in the target space of the auxiliary scalar fields φ^i $(i=1,2)$.

We proceed now with the construction of a new string action that employs the integration measure $d^2\sigma\,\Phi$ (2) instead of the usual $d^2\sigma\sqrt{-\gamma}$. When considering the types of actions we can have under these circumstances, the first one that comes to mind is the straightforward generalization of the Polyakov-type action (1):

$$S_1=-\frac{1}{2}\int d^2\sigma\,\Phi(\varphi)\gamma^{ab}\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}(X). \qquad (3)$$

Notice that multiplying S_1 by a constant, before boundary or initial conditions are specified, is a meaningless operation since such a constant can be absorbed in a redefinition of the measure fields φ^i $(i=1,2)$ that appear in $\Phi(\varphi)$ (2).

The form (3) is, however, not a satisfactory choice for a string action because the variation of S_1 with respect to γ^{ab} leads to the rather strong condition

*Electronic address: guendel@bgumail.bgu.ac.il
†Electronic address: alexk@bgumail.bgu.ac.il
‡Electronic address: nissimov@inrne.bas.bg
§Electronic address: svetlana@inrne.bas.bg

$$\Phi(\varphi)\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X)=0. \tag{4}$$

If $\Phi\neq 0$, it means that $\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X)=0$, i.e., it means that the metric induced on the string world-sheet vanishes, which is clearly not an acceptable dynamics. Alternatively, if $\Phi=0$, no further information is available—also an undesirable situation.

The situation may be improved by introducing an external antisymmetric tensor gauge field $B_{\mu\nu}(X)$. Then, instead of Eq. (3), we have to consider the action

$$S_2=-\int d^2\sigma\,\Phi(\varphi)\left[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) + \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}}\partial_a X^\mu \partial_b X^\nu B_{\mu\nu}(X)\right], \tag{5}$$

where $\varepsilon^{01}=-\varepsilon^{10}=1$ and $\varepsilon^{00}=\varepsilon^{11}=0$. Varying Eq. (5) with respect to γ^{ab}, we get (if $\Phi\neq 0$)

$$\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}+\gamma_{ab}\frac{\varepsilon^{cd}}{4\sqrt{-\gamma}}\partial_c X^\mu \partial_d X^\nu B_{\mu\nu}=0. \tag{6}$$

Contracting the latter equation with γ^{ab} we see that

$$\frac{\varepsilon^{cd}}{2\sqrt{-\gamma}}\partial_c X^\mu \partial_d X^\nu B_{\mu\nu}=-\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}. \tag{7}$$

Inserting relation (7) into Eq. (6) we obtain

$$\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}-\frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu G_{\mu\nu}=0, \tag{8}$$

which coincides with the form of the string equations of motion corresponding to the Polyakov-type action (1). However, the action (5) is not fully satisfactory since it produces Eq. (6) as an additional constraint.

To make further progress and at the same time to show that one can avoid the need to incorporate an external field, it is important to notice that terms in the action of the form

$$S=\int d^2\sigma\,\sqrt{-\gamma}\,L, \tag{9}$$

which do not contribute to the equations of motion of the standard closed string, i.e., such that $\sqrt{-\gamma}L$ is a total derivative, may yield nontrivial contributions when we consider the counterparts of Eq. (9) of the form

$$S=\int d^2\sigma\,\Phi(\varphi)L. \tag{10}$$

This is so because if $\sqrt{-\gamma}L$ is a total divergence, ΦL in general is not.

The above fact is indeed crucial. For example, let us consider the modified-measure string theory with an additional intrinsic $(1+1)$-dimensional scalar curvature term

$$S_{curv}=-\int d^2\sigma\,\Phi(\varphi)R, \tag{11}$$

which now *is not a topological* term in contrast to $\int d^2\sigma\sqrt{-\gamma}R$ in the ordinary string theory with the regular world-sheet integration measure. According to Refs. [5], where modified-measure gravity theories in higher dimensions $D>2$ have been explored, we know that in order to achieve physically interesting results one has to proceed in the first order formalism—employing either the affine connection or the spin connection. In the present paper we will restrict ourselves by exploring the spin connection formalism only. This means that the independent dynamical degrees of freedom are the zweibein $e_a^{\bar a}$, the spin connection $\omega_a^{\bar a\bar b}$ ($\bar a=0,1$ are tangent "Lorentz" indexes), and the auxiliary scalar fields φ^i entering the new integration measure density $\Phi(\varphi)$ (2).

We will use the notation $\gamma^{ab}=e^a_{\bar a}e^b_{\bar b}\eta^{\bar a\bar b}$; the scalar curvature of the spin conection is $R(\omega,e)=e^{a\bar a}e^{b\bar b}R_{ab\bar a\bar b}(\omega)$ where

$$R^{\bar a\bar b}_{ab}(\omega)=\partial_a\omega_b^{\bar a\bar b}+\omega_a^{\bar a\bar c}\omega_{b\bar c}^{\;\;\bar b}-(a\leftrightarrow b). \tag{12}$$

Notice now that in $D=2$

$$\omega_a^{\bar a\bar b}=\omega_a\varepsilon^{\bar a\bar b}, \tag{13}$$

where ω_a is a vector field. Therefore, we get for the scalar curvature

$$R(\omega)=\frac{\varepsilon^{ab}}{2\sqrt{-\gamma}}(\partial_a\omega_b-\partial_b\omega_a). \tag{14}$$

We conclude that the vector field ω_a, as a geometrical object associated with the spin connection, can be treated as an Abelian gauge field A_a living on the world-sheet.

Thus, let us consider an Abelian gauge field A_a defined on the world-sheet of the string, in addition to the measure-density fields φ^i that appear in $\Phi(\varphi)$ (2), the usual Riemannian metric γ_{ab}, and the string coordinates X^μ. We can then construct the following nontrivial contribution to the action of the form:

$$S_{gauge}=\frac{1}{2}\int d^2\sigma\,\Phi(\varphi)\frac{\varepsilon^{ab}}{\sqrt{-\gamma}}F_{ab}(A),\quad F_{ab}=\partial_a A_b-\partial_b A_a. \tag{15}$$

Therefore, the total action to be considered now is $S_{\rm string}=S_2+S_{gauge}$, reading explicitly

$$S_{\rm string}=-\int d^2\sigma\,\Phi(\varphi)\left[\frac{1}{2}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}+\frac{\varepsilon^{ab}}{2\sqrt{-\gamma}}[\partial_a X^\mu\partial_b X^\nu B_{\mu\nu}-F_{ab}(A)]\right]\equiv-\int d^2\sigma\,\Phi(\varphi)L. \tag{16}$$

The properties of this model and some of its generalizations will be studied in the following sections.

The action (16) is invariant under a set of diffeomorphisms in the target space of the measure-density fields φ^i combined with a conformal (Weyl) transformation of the metric γ_{ab}, namely,

$$\varphi^i\to\varphi'^i=\varphi'^i(\varphi)\quad\text{so that}\quad\Phi\to\Phi'=J\Phi, \tag{17}$$

where $J=\det\|\partial\varphi'^j/\partial\varphi^j\|$ is the Jacobian of the transformation (17), and

$$\gamma_{ab}\to\gamma'_{ab}=J\gamma_{ab}. \tag{18}$$

In what follows we will refer to the set of transformations (17), (18) as Φ*-extended two-dimensional Weyl transformations* and, accordingly, to the action (16) as being Φ-extended Weyl invariant. Notice also that the spin-curvature term [Eq. (11) with R as in Eq. (14)] is also Φ-extended Weyl invariant (Φ-extended Weyl transformations do not affect the spin connection).

The combination $(\varepsilon^{ab}/\sqrt{-\gamma})F_{ab}$ is a genuine scalar. In two dimensions it is proportional to $\sqrt{F_{ab}F^{ab}}$. In the non-Abelian case one can consider terms in the action of the form $\Phi\sqrt{{\rm Tr}(F_{ab}F^{ab})}$, the latter being a Φ-extended Weyl-invariant object [$\sqrt{Tr(F_{ab}F^{ab})}$ is also a genuine scalar]. This model will be studied in Sec. V below.

To demonstrate some general features of the theory, we will first follow the Lagrangian formalism for solution of the modified-measure string model (16) explored in Refs. [3]. Variation of the action (16) with respect to φ^i yields the equations (here we set $B_{\mu\nu}=0$ for simplicity)

$$\varepsilon^{ab}\partial_b\varphi^i\partial_a\left(\gamma^{cd}\partial_c X^\mu\partial_d X^\nu G_{\mu\nu}(X)-\frac{\varepsilon^{cd}}{\sqrt{-\gamma}}F_{cd}\right)=0. \tag{19}$$

If $\det||\varepsilon^{ab}\partial_b\varphi^i||\neq0$ meaning $\Phi(\varphi)\neq0$, then we conclude that all the derivatives of the quantity inside the parentheses in Eq. (19) must vanish, i.e., this, quantity must equal a certain constant M which will be determined later on:

$$\gamma^{cd}\partial_c X^\mu\partial_d X^\nu G_{\mu\nu}(X)-\frac{\varepsilon^{cd}}{\sqrt{-\gamma}}F_{cd}=M. \tag{20}$$

The equations of motion of the gauge field A_a tell us about how the string tension appears as an integration constant. Indeed, these equations are

$$\varepsilon^{ab}\partial_b\left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\right)=0, \tag{21}$$

which can be integrated to yield a *spontaneously induced* string tension:

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}}={\rm const}\equiv T. \tag{22}$$

Notice that Eq. (22) is perfectly consistent with the Φ-extended Weyl symmetry (17), (18). Equation (20), on the other hand, is consistent with the Φ-extended Weyl symmetry only if $M=0$. We will see in the next section that the equations of motion indeed imply that $M=0$. In the case of higher-dimensional *p*-branes, unlike the string case, the corresponding equations of motion will require a *nonvanishing* constant value of M [cf. Eq. (49) below].

Let us turn our attention to the equations of motion derived from the variation of Eq. (16) with respect to γ^{ab}:

$$\Phi(\varphi)\left(\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}(X)-\frac{1}{2}\gamma_{ab}\frac{\varepsilon^{cd}}{\sqrt{-\gamma}}F_{cd}\right)=0. \tag{23}$$

Solving the constraint Eq. (20) for $(\varepsilon^{cd}/\sqrt{-\gamma})F_{cd}$ and inserting the result back into Eq. (23) we obtain [provided $\Phi(\varphi)\neq0$]

$$\left(\partial_a X^\mu\partial_b X^\nu-\frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu\partial_d X^\nu\right)G_{\mu\nu}(X)+\frac{1}{2}\gamma_{ab}M=0. \tag{24}$$

Multiplying the above equation by γ^{ab} and summing over a,b, we find that $M=0$, i.e., Eqs. (24) with $M=0$ are exactly of the form of Eqs. (8) coming from the standard Polyakov-type action (1) (recall also that it is only $M=0$ which is consistent with the Φ-extended Weyl invariance). After Eq. (22) is used, the equations obtained from the variation of the action (16) with respect to X^μ are seen to be exactly the same as those obtained from the usual Polyakov-type action as well.

II. BOSONIC STRINGS WITH A MODIFIED MEASURE: CANONICAL APPROACH

It is instructive to study the modified-measure string model (16) also within the framework of the canonical Hamiltonian formalism.

Before proceeding let us note that we can extend the model (16) by putting pointlike charges on the string world-sheet which interact with the world-sheet gauge field A_a:

$$S=S_{\rm string}-\sum_i e_i\int d\tau A_0(\tau,\sigma_i). \tag{25}$$

For the canonical momenta of φ^i,A_1,X^μ we obtain [using the short-hand notation L from Eq. (16)]

$$\pi_i^{\varphi} = -\varepsilon_{ij}\partial_\sigma \varphi^j L, \quad \pi_{A_1} \equiv E = \frac{\Phi(\varphi)}{\sqrt{-\gamma}}; \tag{26}$$

$$\mathcal{P}_\mu = \Phi(\varphi)\Bigg[-(\gamma^{00}\dot{X}^\nu + \gamma^{01}\partial_\sigma X^\nu) G_{\mu\nu} - \frac{1}{\sqrt{-\gamma}}\partial_\sigma X^\nu B_{\mu\nu}\Bigg]. \tag{27}$$

Note particularly the second Eq. (26), showing that the ratio of the modified and the usual Riemannian integration-measure densities has the physical meaning of an *electric field strength* on the world-sheet.[1]

We have also the following primary constraints:

$$\pi_{A_0} = 0, \quad \pi_{\gamma^{ab}} = 0, \quad \partial_\sigma \varphi^i \pi_i^{\varphi} = 0, \tag{28}$$

where the last constraint follows directly from the first Eq. (26). From Eqs. (26), (27) we can express the velocities in terms of the canonical coordinates and momenta as follows:

$$\dot{X}^\mu \equiv \dot{X}^\mu(\cdots) = -\frac{G^{\mu\nu}}{\sqrt{-\gamma}\gamma^{00}}\left(\frac{\mathcal{P}_\nu}{E} + \partial_\sigma X^\lambda B_{\nu\lambda}\right) - \frac{\gamma^{01}}{\gamma^{00}}\partial_\sigma X^\mu, \tag{29}$$

$$\begin{aligned}\dot{A}_1 &\equiv \dot{A}_1(\cdots) \\ &= \partial_\sigma A_0 - \sqrt{-\gamma}\frac{\pi_2^{\varphi}}{\partial_\sigma \varphi^1} + \dot{X}^\mu(\cdots)\partial_\sigma X^\nu B_{\mu\nu} \\ &\quad + \sqrt{-\gamma}\bigg(\frac{1}{2}\gamma^{00}\dot{X}^\mu(\cdots)\dot{X}^\nu(\cdots) + \gamma^{01}\dot{X}^\mu(\cdots)\partial_\sigma X^\nu \\ &\quad + \frac{1}{2}\gamma^{11}\partial_\sigma X^\mu \partial_\sigma X^\nu\bigg) G_{\mu\nu}.\end{aligned} \tag{30}$$

In Eq. (30) we used the shorthand notation $\dot{X}^\mu(\cdots)$ defined in Eq. (29). Since the original Lagrangian $\mathcal{L}$ in Eq. (16) is homogeneous of first order with respect to $\dot{\varphi}^i$ we have $\pi_i^{\varphi}\dot{\varphi}^i - \mathcal{L} = 0$ and, therefore, the canonical Hamiltonian reads

$$\begin{aligned}\mathcal{H} = \mathcal{P}_\mu \dot{X}^\mu(\cdots) + E\dot{A}_1(\cdots) = -\frac{1}{\sqrt{-\gamma}\gamma^{00}}\frac{1}{2}\left[\frac{G^{\mu\nu}}{E}(\mathcal{P}_\mu + E\partial_\sigma X^{\mu'} B_{\mu\mu'})(\mathcal{P}_\nu + E\partial_\sigma X^{\nu'} B_{\nu\nu'}) + EG_{\mu\nu}\partial_\sigma X^\mu \partial_\sigma X^\nu\right] \\ + \frac{\gamma^{01}}{\gamma^{00}}\mathcal{P}_\mu \partial_\sigma X^\mu + E\partial_\sigma A_0 - E\sqrt{-\gamma}\frac{\pi_2^{\varphi}}{\partial_\sigma \varphi^1} + \sum_i e_i \delta(\sigma - \sigma_i) A_0,\end{aligned} \tag{31}$$

where we used the expressions for the velocities as functions of the canonical coordinates and momenta (29), (30) and we also included the pointlike charge interaction terms from Eq. (25). Commuting of the canonical Hamiltonian (31) with the primary constraints (28) leads to the following secondary constraints:

$$\frac{\pi_2^{\varphi}}{\partial_\sigma \varphi^1} = 0, \quad \partial_\sigma E - \sum_i e_i \delta(\sigma - \sigma_i) = 0, \tag{32}$$

$$\frac{G^{\mu\nu}}{E}(\mathcal{P}_\mu + E\partial_\sigma X^{\mu'} B_{\mu\mu'})(\mathcal{P}_\nu + E\partial_\sigma X^{\nu'} B_{\nu\nu'}) + EG_{\mu\nu}\partial_\sigma X^\mu \partial_\sigma X^\nu = 0, \tag{33}$$

$$\mathcal{P}_\mu \partial_\sigma X^\mu \equiv (\mathcal{P}_\mu + E\partial_\sigma X^\nu B_{\mu\nu})\partial_\sigma X^\mu = 0. \tag{34}$$

In particular, we obtain that the canonical Hamiltonian is a linear combination of constraints only.

The Poisson algebra of the constraints can be computed straightforwardly. First, we observe that the last constraint in Eq. (28) spans a (centerless) Virasoro algebra:

$$\begin{aligned}&\{\partial_\sigma \varphi^i \pi_i^{\varphi}(\sigma), \partial_{\sigma'} \varphi^i \pi_i^{\varphi}(\sigma')\} \\ &= 2\partial_\sigma \varphi^i \pi_i^{\varphi}(\sigma)\partial_\sigma \delta(\sigma - \sigma') + \partial_\sigma(\partial_\sigma \varphi^i \pi_i^{\varphi})\delta(\sigma - \sigma').\end{aligned} \tag{35}$$

The only nontrivial commutator of the latter with the rest of the constraints is

$$\left\{\partial_\sigma \varphi^i \pi_i^{\varphi}(\sigma), \frac{\pi_2^{\varphi}}{\partial_\sigma \varphi^1}(\sigma')\right\} = -\partial_\sigma\left(\frac{\pi_2^{\varphi}}{\partial_\sigma \varphi^1}\right)\delta(\sigma - \sigma'). \tag{36}$$

Therefore, both constraints $\partial_\sigma \varphi^i \pi_i^{\varphi}$ and $\pi_2^{\varphi}/\partial_\sigma \varphi^1$ or, equivalently, their linear combination $\mathcal{F}_1 \equiv \partial_\sigma \varphi^i \pi_i^{\varphi}$ and $\mathcal{F}_2 \equiv \pi^{\varphi_2}/\partial_\sigma \varphi^1 - \pi^{\varphi_1}/\partial_\sigma \varphi^2$, span a closed algebra of first-class constraints, which implies that all auxiliary scalars φ^i entering the modified measure (2) are *pure gauge* degrees of freedom. Let us note that both constraints $\mathcal{F}_{1,2}$ involving the auxiliary scalars act only on the latter:

[1]In analogy with ordinary electrodynamics or Yang-Mills theory the canonically conjugated momentum $\pi_{A_1} \equiv E$ of the spacelike gauge-field component A_1 is by definition the electric field strength. However, unlike the ordinary case E is now *not* proportional to $F_{01}(A)$; see also Sec. V for the non-Abelian case.

$$\delta_1\varphi^i \equiv -\{\mathcal{F}_1(\sigma),\varphi^i(\sigma')\} = \partial_\sigma\varphi^i\,\delta(\sigma-\sigma'),$$

$$\delta_2\varphi^i \equiv -\{\mathcal{F}_2(\sigma),\varphi^i(\sigma')\} = -\varepsilon^{ij}(\partial_\sigma\varphi^j)^{-1}\delta(\sigma-\sigma'), \tag{37}$$

whereas they leave unaffected the rest of the dynamical string degrees of freedom. The same result holds also in the more general case of modified-measure p-brane models.

Next, we observe that the second constraint in Eq. (32) is nothing but a Gauss-law first-class constraint for the world-sheet Abelian gauge field, with E being the corresponding electric field strength. Obviously, E is piecewise constant (with respect to σ) on the world-sheet with jumps at the locations of the pointlike charges:

$$E = E_0 + \sum_i e_i\,\theta(\sigma-\sigma_i). \tag{38}$$

Moreover, since the canonical Hamiltonian (31) does not depend explicitly on A_1, E is conserved (world-sheet time independent).

Finally, the constraints (33), (34), or, more properly, the linear combinations thereof,

$$\mathcal{T}_\pm \equiv \frac{1}{4}G^{\mu\nu}\left(\frac{\mathcal{P}_\mu}{E} \pm (G_{\mu\kappa}\pm B_{\mu\kappa})\partial_\sigma X^\kappa\right) \times\left(\frac{\mathcal{P}_\nu}{E} \pm (G_{\nu\lambda}\pm B_{\nu\lambda})\partial_\sigma X^\lambda\right), \tag{39}$$

span the same first-class constraint algebra of two mutually commuting centerless Virasoro algebras as in the case of the ordinary Polyakov-type string [in the standard case $H_{\mu\nu\lambda}(B)\equiv 3\partial_{[\mu}B_{\nu\lambda]}=0$] provided we identify the constant world-sheet electric field E with the ordinary string tension T.

To summarize so far, we find that the modified-measure string model (16) [or (25)], containing no *ad hoc* dimensionful parameters, produces a *dynamically generated* effective string tension, which is equal to the ratio of the modified and usual Riemannian integration-measure densities, and which has the physical meaning of a world-sheet electric field strength. As a result the dynamical string tension is (piecewise) constant along the string with possible jumps at the locations of attached pointlike charges (see Sec. V C for explicit examples).

III. BOSONIC BRANES WITH A MODIFIED MEASURE

The action of bosonic p-branes with a modified world-volume integration measure reads (cf. [3])

$$S_{p\text{-brane}} = -\int d^{p+1}\sigma\,\Phi(\varphi)\left[\frac{1}{2}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu G_{\mu\nu} + \frac{\varepsilon^{a_1\ldots a_{p+1}}}{(p+1)\sqrt{-\gamma}}\left[\partial_{a_1}X^{\mu_1}\cdots\partial_{a_{p+1}}X^{\mu_{p+1}}B_{\mu_1\ldots\mu_{p+1}} - F_{a_1\ldots a_{p+1}}(A)\right]\right] \equiv -\int d^{p+1}\sigma\,\Phi(\varphi)L; \tag{40}$$

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!}\varepsilon_{i_1\ldots i_{p+1}}\varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1}\varphi^{i_1}\cdots\partial_{a_{p+1}}\varphi^{i_{p+1}} = \frac{1}{p!}\varepsilon_{ij_1\ldots j_p}\varepsilon^{\alpha_1\ldots\alpha_p}\dot{\varphi}^i\partial_{\alpha_1}\varphi^{j_1}\cdots\partial_{\alpha_p}\varphi^{j_p}. \tag{41}$$

Here the following notation is used:

$$\underline{\sigma} \equiv (\sigma^a) \equiv (\sigma^0 \equiv \tau, \sigma^\alpha) \equiv (\tau, \vec{\sigma});$$

$$F_{a_1\ldots\alpha_{p+1}}(A) = (p+1)\partial_{[a_1}A_{a_2\ldots a_{p+1}]}, \tag{42}$$

where $a,b=0,1,\ldots,p$; $\alpha,\beta=1,\ldots,p$; $i,j=1,\ldots,p+1$; $\mu,\nu=0,1,\ldots,D-1$; and $G_{\mu\nu}$ and $B_{\mu_1\ldots\mu_{p+1}}$ denote space-time metric and antisymmetric $(p+1)$-rank tensor external fields, respectively. Also, it is convenient within the Hamiltonian formalism to introduce the following notation:

$$\varepsilon^{\alpha_1\ldots\alpha_p}F_{0\alpha_1\ldots\alpha_p}(A) = \dot{\mathcal{A}} - \partial_\alpha\mathcal{A}_0^\alpha \tag{43}$$

with

$$\mathcal{A} \equiv \varepsilon^{\alpha_1\ldots\alpha_p}A_{\alpha_1\ldots\alpha_p}, \quad \mathcal{A}_0^\alpha \equiv \varepsilon^{\alpha\beta_1\ldots\beta_{p-1}}A_{0\beta_1\ldots\beta_{p-1}}. \tag{44}$$

In analogy with the string case we can put (closed) $(p-1)$-branes on the world-volume of the modified-measure p-brane (40) coupled to the latter via the auxiliary world-volume p-form gauge field $A_{a_1\ldots a_p}$ giving rise to the following additional term in the action (40):

$$S=S_{p\text{-brane}}+S_{(p-1)\text{-brane}},$$

$$S_{(p-1)\text{-brane}}=\sum_i e_i\int d^{p+1}\sigma A_{a_1\ldots a_p}(\underline{\sigma})\int d^p u\frac{1}{p!}\varepsilon^{\alpha_1\cdots\alpha_p}\frac{\partial\sigma_i^{a_1}}{\partial u^{\alpha_1}}\cdots\frac{\partial\sigma_i^{a_p}}{\partial u^{\alpha_p}}\delta^{(p+1)}[\underline{\sigma}-\underline{\sigma}_i(\underline{u})]$$

$$=\sum_i e_i\int d^{p+1}\sigma\,\mathcal{A}_0^{\alpha}(\underline{\sigma})\int d^{p-1}u\,\frac{1}{[(p-1)!]^2}\varepsilon_{\alpha\alpha_1\ldots\alpha_{p-1}}\varepsilon^{m_1\cdots m_{p-1}}\frac{\partial\sigma_i^{\alpha_1}}{\partial u^{m_1}}\cdots\frac{\partial\sigma_i^{\alpha_{p-1}}}{\partial u^{m_{p-1}}}\delta^{(p)}[\vec{\sigma}-\vec{\sigma}_i(\vec{u})]. \tag{45}$$

Here $\underline{u}\equiv(u^0=\tau,u^m)\equiv(\tau,\vec{u})$ with $m=1,\ldots,p-1$ are the world-volume parameters of the pertinent $(p-1)$-branes embedded in the world-volume of the original p-brane via the parameter equations $\underline{\sigma}=\underline{\sigma}_i(\underline{u})$ (and we have chosen the static gauge $\sigma^0\equiv\tau=u^0$ for all of them).[2] Also, in the second equality (45) we have used the notation from (44).

The Lagrangian formalism analysis of the modified-measure p-brane model (without attached lower-dimensional branes) (40) has been performed in [3]. It parallels the analysis of the modified-measure string model (cf. Sec. I) where the analogues of Eqs. (20)–(23) now read (taking for simplicity $B_{\mu_1\ldots\mu_{p+1}}=0$)

$$\frac{1}{2}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}-\frac{\varepsilon^{a_1\cdots a_{p+1}}}{(p+1)\sqrt{-\gamma}}\times F_{a_1\ldots a_{p+1}}(A)=M, \tag{46}$$

$$\varepsilon^{aa_1\cdots a_p}\partial_a\left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\right)=0\rightarrow\frac{\Phi(\varphi)}{\sqrt{-\gamma}}=\text{const}\equiv T, \tag{47}$$

$$\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}-\frac{\gamma_{ab}}{(p+1)\sqrt{-\gamma}}\varepsilon^{a_1\cdots a_{p+1}}\times F_{a_1\ldots a_{p+1}}(A)=0. \tag{48}$$

In Eq. (46) M denotes an arbitrary integration constant which enters the relation between the intrinsic and the induced metrics on the p-brane world-volume that follows from Eqs. (46) and (48):

$$\gamma_{ab}=\frac{p-1}{2M}\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}. \tag{49}$$

Also we have

$$\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}-\frac{\gamma_{ab}}{p+1}\gamma^{cd}\partial_c X^\mu\partial_c X^\nu G_{\mu\nu}=0. \tag{50}$$

[2] In what follows we assume that the $(p-1)$-branes do not intersect with each other on the original p-brane world-volume.

The arbitrariness of M is due to the manifest invariance of the modified-measure p-brane action (40) under the following global scale symmetry [3]:

$$\varphi^i\rightarrow\lambda_i\varphi^i,\quad \gamma_{ab}\rightarrow\left(\prod_i\lambda_i\right)\gamma_{ab},$$
$$A_{a_1\ldots a_p}\rightarrow\left(\prod_i\lambda_i\right)^{(p-1)/2}A_{a_1\ldots a_p}, \tag{51}$$

which can be used to fix the value of M, e.g., $M=\frac{1}{2}(p-1)$. Note that the "boundary" term (45) is not invariant under the scale symmetry (51), unless we simultaneously rescale the "charge" coupling constants e_i. Moreover, unlike the string case there is no analogue of the Φ-extended Weyl symmetry (17), (18) for the modified-measure p-brane model (40). The reason is that for $p\geqslant 2$ the standard measure density $\sqrt{-\gamma}$ transforms differently $\sqrt{-\gamma}\rightarrow[J(\varphi)]^{(p+1)/2}\sqrt{-\gamma}$ from the modified measure density $\Phi(\varphi)$ (17) (cf. Refs. [7]).

The canonical Hamiltonian treatment of the p-brane model (40) with attached $(p-1)$-branes on its world-volume similarly follows the same steps as the canonical treament of the modified-measure string model in the previous section. For the canonical momenta of $\varphi^i,\mathcal{A},X^\mu$ we have [using the shorthand notation L from Eq. (40)]

$$\pi_i^\varphi=-\frac{1}{p!}\varepsilon_{ij_1\cdots j_p}\varepsilon^{\alpha_1\cdots\alpha_p}\partial_{\alpha_1}\varphi^{j_1}\ldots\partial_{\alpha_p}\varphi^{j_p}L,$$

$$\pi_{\mathcal{A}}\equiv\mathcal{E}=\frac{\Phi(\varphi)}{\sqrt{-\gamma}}, \tag{52}$$

$$\mathcal{P}_\mu=\Phi(\varphi)\left[-(\gamma^{00}\dot{X}^\nu+\gamma^{0\alpha}\partial_\alpha X^\nu)G_{\mu\nu}-\frac{\varepsilon^{\alpha_1\cdots\alpha_p}}{\sqrt{-\gamma}}\partial_{\alpha_1}X^{\nu_1}\cdots\partial_{\alpha_p}X^{\nu_p}B_{\mu\nu_1\ldots\nu_p}\right]. \tag{53}$$

Also, similarly to Eq. (28) we have the following primary constraints:

$$\pi_{\mathcal{A}_0^\alpha}=0,\quad \pi_{\gamma^{ab}}=0,\quad \partial_\alpha\varphi^i\pi_i^\varphi=0, \tag{54}$$

where the last Virasoro-like constraints follow directly from the first Eq. (52).

At this point it is convenient to reexpress the world-volume Riemannian metric γ_{ab} in terms of its purely space-like part $\gamma_{\alpha\beta}$ and the associated shift vector N^α and lapse function N (see, e.g., [8]):

$$\gamma_{00} = -N^2\overline{\gamma} + \overline{\gamma}_{\alpha\beta}N^\alpha N^\beta,$$

$$\gamma_{0\alpha} = \overline{\gamma}_{\alpha\beta}N^\beta, \quad \overline{\gamma}_{\alpha\beta} = \gamma_{\alpha\beta}, \tag{55}$$

where $\overline{\gamma} = \det\|\gamma_{\alpha\beta}\|$. In particular, $\sqrt{-\gamma} = N\overline{\gamma}$.

Using Eqs. (52), (53) and the notation (55) we find the following canonical Hamiltonian [cf. Eq. (31)] :

$$\mathcal{H} = \frac{N}{2}\left(\frac{G^{\mu\nu}}{\mathcal{E}}\widetilde{\mathcal{P}}_\mu\widetilde{\mathcal{P}}_\nu + \mathcal{E}\overline{\gamma}\gamma^{\alpha\beta}G_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu\right) + \mathcal{E}N\,F(\pi^\varphi,\partial\varphi) - N^\alpha\widetilde{\mathcal{P}}_\mu\partial_\alpha X^\mu + \mathcal{E}\partial_\alpha\mathcal{A}_0^\alpha + [(p-1)-\text{brane terms}] \tag{56}$$

with the shorthand notation

$$\widetilde{\mathcal{P}}_\mu \equiv \mathcal{P}_\mu + \mathcal{E}\varepsilon^{\alpha_1\cdots\alpha_p}\partial_{\alpha_1}X^{\mu_1}\ldots\partial_{\alpha_p}X^{\mu_p}B_{\mu\mu_1\ldots\mu_p},$$

$$F(\pi^\varphi,\partial\varphi) \equiv (p-1) \times !\sum_{i=2}^{p+1}\frac{\pi_i^\varphi}{\varepsilon_{ij_1\ldots j_p}\varepsilon^{\alpha_1\cdots\alpha_p}\partial_{\alpha_1}\varphi^{j_1}\cdots\partial_{\alpha_p}\varphi^{j_p}}, \tag{57}$$

and where the last terms in Eq. (56) come from Eq. (45).

Commuting the canonical Hamiltonian (56) with the primary constraints (54) [where upon using the notations (55) we have $\pi_N = 0, \pi_{N^\alpha} = 0, \pi_{\overline{\gamma}^{\alpha\beta}} = 0$ instead of $\pi_{\gamma^{ab}} = 0$] we obtain a set of secondary constraints. Using the Poisson-bracket relation

$$\{\partial_\alpha\varphi^i\pi_i^\varphi(\vec{\sigma}), F(\pi^\varphi,\partial\varphi)(\vec{\sigma}\,')\} = -\delta(\vec{\sigma}-\vec{\sigma}\,')\partial_a F(\pi^\varphi,\partial\varphi)(\vec{\sigma}) \tag{58}$$

we get the following secondary constraint:

$$\partial_\alpha F(\pi^\varphi,\partial\varphi) = 0 \rightarrow F(\pi^\varphi,\partial\varphi) = -2M \equiv \text{const}, \tag{59}$$

where M is an arbitrary constant [it is the Hamiltonian counterpart of the arbitrary integration constant M appearing within the Lagrangian treatment; cf. Eq. (46)]. Once again, as in the string case, we find that the Virasoro-like constraints $\partial_\alpha\varphi^i\pi_i^\varphi$ together with $F(\pi^\varphi,\partial\varphi) + 2M = 0$ [the latter being defined in Eq. (57)] form a closed algebra of first-class constraints, implying that the auxiliary scalar fields φ^i are pure gauge degrees of freedom.

Next, commuting Eq. (56) with $\pi_{\mathcal{A}_0^\alpha}$ yields

$$\partial_\alpha\mathcal{E}(\underline{\sigma}) + \sum_i e_i\int d^{p-1}u\frac{1}{[(p-1)!]^2}\varepsilon_{\alpha\alpha_1\ldots\alpha_{p-1}}\varepsilon^{m_1\cdots m_{p-1}} \times\frac{\partial\sigma_i^{\alpha_1}}{\partial u^{m_1}}\cdots\frac{\partial\sigma_i^{\alpha_{p-1}}}{\partial u^{m_{p-1}}}\delta^{(p)}[\vec{\sigma}-\vec{\sigma}_i(\vec{u})] = 0, \tag{60}$$

which is the p-brane analogue of the "Gauss"-law constraint in the string case [second Eq. (32)]. Further, since the canonical Hamiltonian (56) does not depend explicitly on $\mathcal{A}$ (44) (canonically conjugate to $\mathcal{E}$), the p-brane "electric" field strength $\mathcal{E}$ is conserved (world-volume time independent) and as long as it obeys the generalized "Gauss law" on the world-volume Eq. (60), $\mathcal{E}$ is also a world-volume *piecewise* constant field with jumps along the normals equal to the "charge" e_i when crossing the world-hypersurface of the ith $(p-1)$-brane.

The rest of the secondary constraints reads

$$\frac{G^{\mu\nu}}{\mathcal{E}}\widetilde{\mathcal{P}}_\mu\widetilde{\mathcal{P}}_\nu + \mathcal{E}\overline{\gamma}\gamma^{\alpha\beta}G_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu - 4M\overline{\gamma}\mathcal{E} = 0, \tag{61}$$

$$\widetilde{\mathcal{P}}_\mu\partial_\alpha X^\mu = 0,$$

$$\partial_\alpha X^\mu\partial_\beta X^\nu G_{\mu\nu} - \frac{2M}{p-1}\overline{\gamma}_{\alpha\beta} = 0. \tag{62}$$

We now observe that for the special choice $M = \frac{1}{2}(p-1)$, and provided we identify the "electric" field strength $\mathcal{E}$ as a dynamical brane tension T, the constraints (61), (62) coincide with the secondary constraints within the Hamiltonian treatment of the usual Polyakov-like p-brane (the latter together with the primary constraints forms a mixture of first-class and second-class constraints).

Thus, we conclude from Eqs. (56), (57) and (61), (62) that the modified-measure p-brane model (40) possesses, apart from the same brane degrees of freedom as the standard Polyakov-like p-brane, an additional brane degree of freedom $\mathcal{E}$—a world-volume "electric" field strength, which can be identified as a dynamical brane tension and which, according to Eq. (60), may be variable in general.[3]

IV. SUPERSTRINGS WITH A MODIFIED MEASURE

We consider the following Green-Schwarz-type of superstring action with a modified world-sheet integration measure (for the standard Green-Schwarz superstring action, see, e.g. [9]):

[3] More detailed discussion of this issue will appear elsewhere: E. I. Guendelman, A. Kaganovich, E. Nissimov, and S Pacheva, in *Proceedings of the First Conference on Gravity, Astrophysics and Strings at the Black Sea*, edited by P. Fiziev *et al.* (to be published).

$$S_{\rm superstring}=\int d^2\sigma\,\Phi(\varphi)\bigg[-\frac{1}{2}\gamma^{ab}\Pi_a^\mu\Pi_{b\,\mu}+\frac{\varepsilon^{ab}}{\sqrt{-\gamma}}\bigg(\Pi_a^\mu(\theta\sigma_\mu\partial_b\theta)+\frac{1}{2}F_{ab}(A)\bigg)\bigg]\equiv-\int d^2\sigma\,\Phi(\varphi)L \tag{63}$$

with the same notation as in Eqs. (16) and (2) (for simplicity we now take $G_{\mu\nu}=\eta_{\mu\nu},B_{\mu\nu}=0$) and where

$$\Pi_a^\mu\equiv\partial_aX^\mu+i\theta\sigma^\mu\partial_a\theta. \tag{64}$$

Here $\theta\equiv(\theta^\alpha)$ $(\alpha=1,\dots,16)$ denotes a 16-dimensional Majorana-Weyl spinor in the embedding $D=10$ space-time, whereas $\sigma^\mu\equiv((\sigma^\mu)_{\alpha\beta})$ indicate the upper diagonal 16×16 blocks of the 32×32 matrices $\mathcal{C}^{-1}\Gamma^\mu$ with Γ^μ and $\mathcal{C}$ being the $D=10$ Dirac and charge-conjugation matrices, respectively.

The Lagrangian in Eq. (63) is explicitly invariant under space-time supersymmetry transformations

$$\delta_\epsilon\theta=\epsilon,\quad \delta_\epsilon X^\mu=-i(\epsilon\sigma^\mu\theta),$$
$$\delta_\epsilon A_a=i(\epsilon\sigma_\mu\theta)\bigg(\partial_aX^\mu+\frac{i}{3}\theta\sigma^\mu\partial_a\theta\bigg). \tag{65}$$

In particular, the algebra of supersymmetry transformations (65) closes on A_a up to a gauge transformation:

$$\{\delta_{\epsilon_1},\delta_{\epsilon_2}\}A_a=\partial_a\bigg(-\frac{2}{3}(\epsilon_1\sigma^\mu\theta)(\epsilon_2\sigma_\mu\theta)\bigg). \tag{66}$$

Also, the modified-measure superstring action (63) is invariant under the local fermionic kappa symmetry, similarly to the standard Green-Schwarz superstring action [9]:

$$\delta_\kappa\theta=2i\Pi_a^\mu(\sigma_\mu\kappa^a),\quad \delta_\kappa X^\mu=i(\theta\sigma^\mu\delta_\kappa\theta),$$
$$\delta_\kappa A_a=0,$$
$$\delta_\kappa(\sqrt{-\gamma}\gamma^{ab})=-8\sqrt{-\gamma}\bigg(\gamma^{ac}+\frac{\varepsilon^{ac}}{\sqrt{-\gamma}}\bigg)(\kappa^b\partial_c\theta),$$
$$\delta_\kappa\varphi^i=0, \tag{67}$$

where $\kappa^a=\frac{1}{2}(\gamma^{ab}+\varepsilon^{ab}/\sqrt{-\gamma})\kappa_b$ is the self-dual fermionic local gauge parameter. In particular, let us note that $\delta_\kappa(\Phi(\varphi)/\sqrt{-\gamma})=0$.

Let us note that the action (63) bears a resemblance to the modified Green-Schwarz superstring action proposed by Siegel [10] provided we replace the modified integration measure density $\Phi(\varphi)$ with the ordinary one $\sqrt{-\gamma}$ and provided we redefine the auxiliary gauge field A_a as a fermionic bilinear composite $A_a=-i\theta_\alpha\partial_a\phi^\alpha$ (cf. second Ref. [3]) with ϕ indicating Siegel's auxiliary fermionic world-sheet field, which is a space-time spinor similar to θ. However, let us emphasize that our present approach to the modified-measure superstring model (63) is consistently based on a fundamental (noncomposite) gauge field A_a.

For the canonical momenta of $\varphi^i,A_1,X^\mu,\theta$ we have [using the shorthand notation L from Eq. (63) and the definition (64)]

$$\pi_i^\varphi=-\varepsilon_{ij}\partial_\sigma\varphi^jL,\quad \pi_{A_1}\equiv E=\frac{\Phi(\varphi)}{\sqrt{-\gamma}}, \tag{68}$$

$$\mathcal{P}_\mu=\Phi(\varphi)\bigg[-(\gamma^{00}\Pi_{0\mu}-\gamma^{01}\Pi_{1\mu})+\frac{i}{\sqrt{-\gamma}}(\theta\sigma_\mu\theta')\bigg], \tag{69}$$

$$\mathcal{P}_\theta\bigg[-(\gamma^{00}\Pi_{0\,\mu}-\gamma^{01}\Pi_{1\,\mu})-\frac{i}{\sqrt{-\gamma}}X'_\mu\bigg]i\theta\sigma^\mu, \tag{70}$$

where the prime now indicates the derivative ∂_σ. From Eqs. (69), (70) and taking into account the second Eq. (68), we obtain the fermionic primary constraint

$$i\mathcal{D}\equiv\mathcal{P}_\theta-(\mathcal{P}_\mu-E\Pi_{1,\mu})i\theta\sigma^\mu=0. \tag{71}$$

Therefore, we have the following set of primary constraints:

$$\pi_{A_0}=0,\quad \pi_{\gamma^{ab}}=0,\quad \partial_\sigma\varphi^i\pi_i^\varphi=0,\quad \mathcal{D}=0. \tag{72}$$

Now, for the velocities as functions of the canonical coordinate and momenta we get

$$\dot X^\mu+i\theta\sigma^\mu\dot\theta\equiv\Pi_0^\mu(\cdots)=\frac{1}{\sqrt{-\gamma}\gamma^{00}}\bigg(\frac{-\mathcal{P}^\mu}{E}+i\theta\sigma^\mu\theta'\bigg)-\frac{\gamma^{01}}{\gamma^{00}}\Pi_1^\mu \tag{73}$$

$$\begin{aligned}\dot A_1-i\Pi_1^\mu(\theta\sigma_\mu\dot\theta)\equiv\dot{\mathcal{A}}_1(\cdots)&=\partial_\sigma A_0-\sqrt{-\gamma}\frac{\pi_2^\varphi}{\partial_\sigma\varphi^1}\\&+\sqrt{-\gamma}\bigg(\frac{1}{2}\gamma^{00}\Pi_0^\mu(\cdots)\Pi_{0\mu}(\cdots)\\&+\gamma^{01}\Pi_0^\mu(\cdots)\Pi_{1m}\\&+\frac{1}{2}\gamma^{11}\Pi_1^\mu\Pi_{1\,m}\bigg)-i(\theta\sigma_\mu\theta')\Pi_0^\mu(\cdots).\end{aligned} \tag{74}$$

In Eq. (74) we used the shorthand notation $\Pi_0^\mu(\cdots)$ defined in Eq. (73). The canonical Hamiltonian reads

$$\begin{aligned}\mathcal{H}&=\mathcal{P}_\mu\dot X^\mu(\cdots)+\mathcal{P}_\theta\dot\theta(\cdots)+E\dot A_1(\cdots)+i\Lambda_\alpha\mathcal{D}^\alpha\\&=\mathcal{P}_\mu\Pi_0^\mu(\cdots)+E\dot{\mathcal{A}}_1(\cdots)+i\mathcal{D}(\dot\theta(\cdots)-\Lambda).\end{aligned} \tag{75}$$

Here $(\cdots)$ indicate that all velocities are considered as functions of the canonical coordinate and momenta according to Eqs. (73), (74); $\mathcal{D}$ is the fermionic primary constraint (71) and Λ is the corresponding fermionic Lagrange multiplier which is determined from the requirement of the preservation of the constraint $\mathcal{D}$ under the Hamiltonian dynamics by Eq. (75). Inserting in Eq. (75) the expressions (73), (74) we obtain

$$\mathcal{H}=-\frac{1}{\sqrt{-\gamma}\gamma^{00}}\frac{1}{2}\left[\frac{1}{E}[\mathcal{P}^{\mu}-iE(\theta\sigma^{\mu}\theta')]\times[\mathcal{P}_{\mu}-iE(\theta\sigma_{\mu}\theta')]+E\Pi_{1}^{\mu}\Pi_{1\mu}\right]+\frac{\gamma^{01}}{\gamma^{00}}[\mathcal{P}_{\mu}-iE(\theta\sigma_{\mu}\theta')]\Pi_{1}^{\mu}+i\Lambda\mathcal{D}+E\partial_{\sigma}A_{0}-E\sqrt{-\gamma}\frac{\pi_{2}^{\varphi}}{\partial_{\sigma}\varphi^{1}}. \tag{76}$$

Commuting of the canonical Hamiltonian (76) with the primary constraints (72) leads to the following secondary constraints:

$$\frac{\pi_{2}^{\varphi}}{\partial_{\sigma}\varphi^{1}}=0,\quad \partial_{\sigma}E=0(\text{``Gauss law''}), \tag{77}$$

$$\mathcal{T}_{+}\equiv\frac{1}{4}\left[\frac{\mathcal{P}}{E}+E(X'-2i\theta\sigma\theta')\right]^{2}-i\theta'\mathcal{D}=0,$$

$$\mathcal{T}_{-}\equiv\frac{1}{4}\left(\frac{\mathcal{P}}{E}-EX'\right)^{2}. \tag{78}$$

Therefore, as in the purely bosonic case we conclude that the canonical Hamiltonian is a linear combination of constraints only.

As in the bosonic case, the constraints involving the auxiliary scalar fields φ^{i} span the same Poisson-bracket algebra (35), (36) and, therefore, the auxiliary scalars are again pure gauge degrees of freedom. The rest of the constraint algebra is the same as in the case of the standard Green-Schwarz formulation provided (in full analogy with the purely bosonic case) we identify the world-sheet "electric" field strength E as dynamically generated string tension T.

Let us recall that the 16-component fermionic spinor constraint $\mathcal{D}$ (71) (which is identical to the corresponding fermionic constraint in the standard Green-Schwarz superstring model) contains a Lorentz noncovariant mixture of eight first-class constraints [Hamiltonian generators of kappa symmetry transformations (67)] and eight second-class constraints. To solve the problem of super-Poincaré covariant quantization of the modified-measure Green-Schwarz superstring (63) we can, therefore, employ the same kind of reformulation of (63) as in the standard Green-Schwarz case, which has been proposed in Refs. [11] involving a special set of additional auxiliary pure gauge bosonic space-time spinor and vector fields which are simultaneously world-sheet scalars ("harmonic" variables). For recent developments on this subject, see [12] and references therein.

V. STRINGS WITH "Φ-EXTENDED WEYL INVARIANT" ACTION FOR NON-ABELIAN WORLD-SHEET GAUGE FIELD

A. The regular-measure version of the theory

As it is well known, in four space-time dimensions the standard gauge field action proportional to $\int\sqrt{-g}d^{4}x\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ is invariant under transformations $g_{\mu\nu}\rightarrow\Omega^{2}(x)g_{\mu\nu}$, i.e., it is conformally invariant. In $D=2$, the appropriate conformally invariant action, provided we use the standard measure $\sqrt{-\gamma}$, would be

$$\int d^{2}\sigma\sqrt{-\gamma}\sqrt{\frac{1}{2}\mathrm{Tr}[F_{ab}(A)F_{cd}(A)]\gamma^{ac}\gamma^{bd}}=\int d^{2}\sigma\sqrt{\mathrm{Tr}[F_{01}(A)F_{01}(A)]}, \tag{79}$$

where

$$F_{ab}(A)=\partial_{a}A_{b}-\partial_{b}A_{c}+i[A_{a},A_{b}] \tag{80}$$

is a non-Abelian world-sheet gauge field strength and we have used $F_{ab}(A)=-\varepsilon_{ab}F_{01}(A)$. As we see, the action (79) is not only independent of the conformal factor in the metric, but also it is totally metric independent, i.e., the $D=2$ "square-root Yang-Mills" model (79) is topological in the same sense as, e.g., the $D=3$ Chern-Simons model. Because of this fact the string and gauge degrees of freedom turn out to be decoupled.

To see that such a theory does not lead to a well defined dynamics and instead a modified-measure version of Eq. (79) is necessary, we consider first the equations of motion that result from Eq. (79). Variation with respect to gauge fields A_{a} yields

$$\nabla_{a}\left(\frac{F_{01}}{\sqrt{\mathrm{Tr}(F_{01}F_{01})}}\right)=0 \tag{81}$$

or, equivalently

$$\nabla_{a}F_{01}-F_{01}\frac{\mathrm{Tr}(F_{01}\nabla_{a}F_{01})}{\mathrm{Tr}(F_{01}F_{01})}=0, \tag{82}$$

which in turn are equivalent to the equations

$$\nabla_{a}F_{01}=\partial_{a}fF_{01} \tag{83}$$

with $f\equiv f(\tau,\sigma)$ being an arbitrary colorless world-sheet scalar field. The general solution of Eq. (83) reads

$$F_{01}=G^{-1}e^{f(\tau,\sigma)}\mathcal{M}_{0}G, \tag{84}$$

$$A_{0}=G^{-1}\left(-\mathcal{M}_{0}\int^{\sigma}d\sigma' e^{f(\tau,\sigma')}\right)G-iG^{-1}\partial_{\tau}G,$$

$$A_1 = -iG^{-1}\partial_\sigma G, \tag{85}$$

where G is an arbitrary (τ,σ)-dependent element of the gauge group (reflecting the gauge freedom) whereas $\mathcal{M}_0$ is an arbitrary constant element of the corresponding Lie algebra.

Thus, we see that in the $D=2$ "square-root Yang-Mills" action (79) there is an additional freedom in equations of motion (beyond the usual non-Abelian gauge symmetry) which is manifested in the appearance of the arbitrary (not determined by the dynamics) world-sheet scalar field $f(\tau,\sigma)$ in Eqs. (83)–(85).

This can be equivalently understood from the canonical Hamiltonian point of view; namely, one can show that the canonical Hamiltonian of the $D=2$ "square-root Yang-Mills" model (79) is a linear combination of first-class constraints only in contrast to the ordinary Yang-Mills case:

$$\mathcal{H} = \mathrm{Tr}\{\mathcal{E}(\partial_\sigma A_0 + i[A_1, A_0])\} + \Lambda_0 \pi_{A_0} + \frac{\Lambda}{2}(\mathrm{Tr}\,\mathcal{E}^2 - 1), \tag{86}$$

where π_{A_0} and $\mathcal{E} = F_{01}/\sqrt{\mathrm{Tr}(F_{01}F_{01})}$ are the canonical momenta of A_0 and A_1, respectively, and where Λ_0, Λ are the corresponding Lagrange multipliers. Notice the appearance of the third first-class constraint term in Eq. (86) instead of the standard nonconstraint term $\frac{1}{2}\mathrm{Tr}\,\mathcal{E}^2$. Moreover, the total number of first-class constraints in Eq. (86) exceeds the number of the underlying degrees of freedom.

B. Modified-measure version: The case of closed strings without charges

We will now see that the modified-measure version of non-Abelian world-sheet gauge fields has a well defined dynamics (in contrast to the regular-measure case of the previous subsection) provided that the theory possesses the Φ-extended Weyl symmetry. We consider the following non-Abelian generalization of the original bosonic string action with a modified measure (16) (now we take for simplicity $G_{\mu\nu} = \eta_{\mu\nu}$ and $B_{\mu\nu} = 0$):

$$S = -\int d^2\sigma\, \Phi(\varphi)\left[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu - \sqrt{\frac{1}{2}\mathrm{Tr}[F_{ab}(A)F_{cd}(A)]\gamma^{ac}\gamma^{bd}}\right]$$
$$= -\int d^2\sigma\, \Phi(\varphi)\left[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu - \frac{1}{\sqrt{-\gamma}}\sqrt{\mathrm{Tr}[F_{01}(A)F_{01}(A)]}\right] \equiv -\int d^2\sigma\, \Phi(\varphi) L, \tag{87}$$

where $F_{ab}(A)$ is the non-Abelian world-sheet gauge field strength as in Eq. (80).

Similar to what we saw in Sec. I, the variation with respect to the measure Φ degrees of freedom φ^i leads to the equation (provided that $\Phi \neq 0$)

$$\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu - \frac{1}{\sqrt{-\gamma}}\sqrt{\mathrm{Tr}(F_{01}F_{01})} = M. \tag{88}$$

Varying the action (87) with respect to γ^{ab} we get

$$\partial_a X^\mu \partial_b X_\mu - \frac{1}{\sqrt{-\gamma}}\gamma_{ab}\sqrt{\mathrm{Tr}(F_{01}F_{01})} = 0. \tag{89}$$

Contracting this equation with γ^{ab} and comparing with Eq. (88) we conclude that again, similar to what was shown in the simpler model of Sec I, $M=0$ and we obtain finally

$$\frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu = \sqrt{\mathrm{Tr}(F_{01}F_{01})}, \tag{90}$$

$$\partial_a X^\mu \partial_b X_\mu - \gamma_{ab}\frac{1}{2}\gamma^{cd}\partial_c X^\mu \partial_c X_\mu = 0. \tag{91}$$

Varying the action (87) with respect to A_a we obtain

$$\nabla_a \mathcal{E} \equiv \partial_a \mathcal{E} + i[A_a, \mathcal{E}] = 0, \tag{92}$$

$$\mathcal{E} \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}\frac{F_{01}}{\sqrt{\mathrm{Tr}(F_{01}F_{01})}}$$

with $\mathcal{E}$ being the non-Abelian electric field strength—the canonically conjugated momentum of A_1. Accordingly, Eq. (92) for $a=1$ represents the non-Abelian Gauss law on the world-sheet. Using Eqs. (92) one can easily show that

$$0 = \mathrm{Tr}(\mathcal{E}\nabla_a \mathcal{E}) = \frac{1}{2}\partial_a(\mathrm{Tr}\,\mathcal{E}^2) = \frac{1}{2}\partial_a\left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\right)^2, \tag{93}$$

i.e., the ratio of the measure densities (the magnitude of the non-Abelian electric field strength), which plays the role of a dynamically generated string tension, is again constant: $|\mathcal{E}| \equiv |\Phi(\varphi)/\sqrt{-\gamma}| = \mathrm{const}$. The equations of motion (92), upon using this fact, coincide with the Eqs. (82) [Eqs. (83)–(85) similarly hold]. However, in contrast to the regular-measure version of the theory, now in the context of the modified-measure model (87) we have Eq. (90), which upon substituting the solution (84) in $\sqrt{\mathrm{Tr}(F_{01}F_{01})} = e^{f(\tau,\sigma)}\sqrt{\mathrm{Tr}\mathcal{M}_0^2}$ completely determines the function $f(\tau,\sigma)$ in terms of the string solution.

C. Charges, strings, and classical mechanism for confinement

Classical treatment of strings in the context of the Polyakov approach (with the regular-measure density $\sqrt{-\gamma}$) allows two possibilities for the string topology: the first one is a closed string where the string tension is a constant all over the string; the second possibility is an open string with end points (and/or *ad hoc* with pointlike charges at the end points).

In the modified-measure string theory there are more possibilities due to the dynamical mechanism of tension generation. In fact, for both cases, i.e., for closed and open strings, one can study models where one or more pointlike charges C_i, in general *non-Abelian* ''color'' ones, are located inside the string.[4] A simple model describing this situation consists of adding to the action (87) the following interaction term:

$$S_{int}=-\sum_i \int \frac{d\sigma^a}{d\tau_i}\mathrm{Tr}(C_i A_a)\, d\tau_i, \tag{94}$$

where τ_i indicate the corresponding proper times. In the simplest case of *static* ''color'' charges C_i localized at the points $\sigma_i, (i=1,2,\ldots)$, Eq. (94) reads

$$S_{int,static}=-\sum_i \mathrm{Tr}C_i \int d\tau A_0(\tau,\sigma_i). \tag{95}$$

The only changes in the equations of motion, compared to the equations of the previous subsection, occur in Eq. (92) which in the axial gauge $(A_1=0)$ takes the form

$$\partial_\sigma \mathcal{E}-\sum_i C_i \delta(\sigma-\sigma_i)=0 \tag{96}$$

with $\mathcal{E}$ as defined in the second Eq. (92).

Let us first consider the solution of the ''Gauss law'' Eq. (96) in the case with two static pointlike (color) charges C_1 and C_2 localized at the points σ_1 and σ_2 with $\sigma_1<\sigma_2$. To get this solution we perform the integration in Eq. (96) over σ from some $\sigma<\sigma_1$ up to some $\sigma>\sigma_2$. Then we obtain

$$\mathcal{E}(\sigma)=\begin{cases}\mathcal{E}_1 & \text{for } \sigma<\sigma_1,\\ \mathcal{E}_2 & \text{for } \sigma_1<\sigma<\sigma_2\\ \mathcal{E}_3 & \text{for } \sigma>\sigma_2.\end{cases} \quad \text{and} \quad \mathcal{E}_2-\mathcal{E}_1=C_1, \quad \mathcal{E}_3-\mathcal{E}_2=C_2, \tag{97}$$

To realize the physical case of such an open string (no periodic boundary conditions in σ are assumed) with finite energy we have to consider a finite string, which is possible only if $\mathcal{E}_1\equiv\mathcal{E}_3\equiv 0$. Then the charges C_1 and C_2 appear to be the end points and it follows from Eq. (97) that

$$C_1+C_2=0 \quad \text{and} \quad \mathcal{E}_2=C_1. \tag{98}$$

Therefore, Eq. (98) becomes the statement for color confinement of the two pointlike charges C_i (''quarks'') in a colorless ''mesonlike state'' as a result of the variable dynamical tension of the string connecting them.

In a similar way one can construct a classical string model for baryons. Let us consider a *closed* string parametrized by $\sigma(0\leqslant\sigma\leqslant 2\pi)$ with three static pointlike color charges C_1, C_2, C_3 localized at the points $\sigma_1,\sigma_2,\sigma_3$, respectively. Then by solving Eq. (96) we obtain for the ''chromoelectric'' field, i.e., the dynamical string tension (92),

$$\mathcal{E}(\sigma)=\begin{cases}\mathcal{E}_{12} & \text{for } \sigma_1=0<\sigma<\sigma_2,\\ \mathcal{E}_{23} & \text{for } \sigma_2<\sigma<\sigma_3,\\ \mathcal{E}_{31} & \text{for } \sigma_3<\sigma<2\pi,\end{cases} \tag{99}$$

where $\mathcal{E}_{12},\mathcal{E}_{23},\mathcal{E}_{31}$ are constants, which implies

$$\mathcal{E}_{12}-\mathcal{E}_{31}=C_1, \quad \mathcal{E}_{23}-\mathcal{E}_{12}=C_2, \quad \mathcal{E}_{31}-\mathcal{E}_{23}=C_3. \tag{100}$$

Summing Eqs. (100) we get

$$C_1+C_2+C_3=0, \tag{101}$$

which means that color confinement appears again, now in the case of a ''baryonlike'' configuration.

Notice that not only the orientations of $\mathcal{E}_{12},\mathcal{E}_{23},\mathcal{E}_{31}$ in color space but also their magnitudes are different in general. The last statement follows from the fact that Eq. (93) does not hold at the points where the charges are located. This means that the charges can be sources of discontinuities of the tension [notice that the second equation in (92) still holds]. This is possible here precisely due to the identification of the string tension with the ratio of the measure densities $\Phi(\varphi)/\sqrt{-\gamma}$ [second Eq. (92)] which is also the magnitude of the pertinent world-sheet ''chromoelectric'' field strength. Because of these properties we may call the above modified-measure string model with a Φ-extended Weyl-invariant non-Abelian world-sheet gauge field action (87) a *''color'' string model*.

The above simple picture of pointlike charge confinement via ''color'' strings can be straightforwardly generalized to the case of higher-dimensional branes; namely, let us consider N nonintersecting ''charged'' closed $(p-1)$-branes liv-

[4]Generically one can consider smooth charge or current distributions along the string. Such more general cases we will study elsewhere; see also the Appendix.

ing on a closed p-brane whose dynamics is governed by the modified-measure brane action (40) and (45). Let us also recall that the dynamically generated brane tension $\mathcal{E}$ [cf. second Eq. (52)] obeys the brane "Gauss-law" constraint Eq. (60). Denoting by $\mathcal{E}_i$ the constant value of $\mathcal{E}$ in the strip on the fixed-time world-hypersurface of the p-brane situated between the $(i-1)$th and the ith "charged" $(p-1)$-branes, we find from Eq. (60)

$$\mathcal{E}_{i+1}=\mathcal{E}_i+e_i\,,\quad i=0,1,\ldots,N \quad \text{with } \mathcal{E}_0\equiv\mathcal{E}_N\,,e_0\equiv e_N\,. \tag{102}$$

Summing up Eqs. (102) we find similarly to the string case that the only possible configuration of static "charged" closed $(p-1)$-branes coupled pairwise via modified-measure p-branes (40) is the zero-charge one.

VI. DISCUSSION AND CONCLUSIONS

We have seen above how modifying the world-sheet (world-volume) measure of integration can significantly affect the implications of string and brane dynamics. First of all, it turns out that, to get an acceptable dynamics, the corresponding string and brane theories need the introduction of an auxiliary world-sheet gauge field (a world-volume p-form tensor gauge field). Furthermore, the tension of the string or brane is not longer a fundamental parameter (i.e., a given *ad hoc* scale): it is dynamically determined as the magnitude of the pertinent gauge field strength and it is proportional to the ratio of the measure densities $\Phi/\sqrt{-\gamma}$. If no charges exist on the world-sheet (world-volume) then for closed strings (branes) the standard Polyakov-type equations are obtained and the Poisson-bracket algebra of the relevant Hamiltonian constraints is the same as that of the standard string (brane) theory. The same result holds also for the modified-measure superstring model.

The string tension is identified as the canonically conjugate momentum of the spatial component of the auxiliary world-sheet gauge potential; therefore, it assumes the role of an "electric" field strength. The latter is shown to obey the "Gauss-law" equation. Thus, in the presence of world-sheet charges, the string tension can change dynamically. The latter becomes possible since the tension, i.e., the "electric" field strength, is proportional to the ratio of the measure densities $\Phi/\sqrt{-\gamma}$. In particular, pointlike charges living on the string can be responsible for discontinuous changes of the string tension. The special case, when the string tension changes from a finite value to zero, can be regarded as the formation of an "edge" on the string or, equivalently, as a new way of formulating open strings. We have shown that similar results hold also for modified-measure theories of p-branes; namely, p-form (tensor gauge) charges living on the p-brane, in particular, lower-dimensional "charged" $(p-1)$-branes lead to a dynamically changing brane tension.

Finally, we studied a conformally (Weyl-)invariant modified-measure string theory with non-Abelian gauge ("square-root Yang-Mills") field living on the string world-sheet called a "color" string. As a result, a simple classical mechanism for "color" confinement of pointlike "color" charges via "color" strings is proposed with the colorlessness of the corresponding composite "hadrons" automatically emerging due to the new dynamics inherent in the modified-measure string model. A similar picture of confinement and colorlessness arises also for systems of "charged" $(p-1)$-branes coupled via modified-measure p-branes.

As a by-product, it is found that a nice geometrical meaning can be given for the auxiliary string world-sheet gauge fields: if these are of the Abelian type, they can represent the world-sheet spin connection associated with the (Abelian in $(1+1)$ dimensions) Lorentz group [see Eq. (14) above].

Notice that world-sheet gauge fields have also been considered in the very interesting work [13]. In that case, however, a Nambu-Goto approach is employed so that the issue of conformal invariance peculiar to the Polyakov formulation is lost.

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APPENDIX: STRINGS WITH A MODIFIED MEASURE COUPLED TO WORLD-SHEET CURRENTS

Let us briefly discuss the case of bosonic strings with a modified world-sheet integration measure coupled to an external space-time dilaton field. The pertinent action reads

$$S=-\int d^2\sigma\,\Phi(\varphi)\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}(X) - R(\omega)\mathcal{U}(X)\Big]\equiv-\int d^2\sigma\,\Phi(\varphi)L \tag{A1}$$

with $R(\omega)$ being the scalar curvature of the $D=2$ spin connection ω_a defined in Eq. (14). Varying Eq. (A1) with respect to ω_a we obtain once again dynamically generated string tension as

$$E\equiv\pi_{\omega_1}=\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\mathcal{U}(X)=\text{const}\equiv T \tag{A2}$$

with π_{ω_1} being the canonically conjugated momentum of ω_1, which brings the action (A1) to the form

$$S=-T\int d^2\sigma\frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu\frac{G_{\mu\nu}(X)}{\mathcal{U}(X)}, \tag{A3}$$

i.e., an action describing string motion in a conformally modified extenal space-time background with $G'_{\mu\nu}(X)=G_{\mu\nu}(X)/\mathcal{U}(X)$. Thus, the model (A1) differs significantly from the ordinary Polyakov-type string coupled to a dilaton:

$$S=-T\int d^2\sigma \frac{1}{2}\sqrt{-\gamma}[\gamma^{ab}\partial_a X^\mu \partial_b X_\mu + R(\gamma)\mathcal{U}(X)]. \tag{A4}$$

Now, let us consider a generalization of the string model (16) describing the coupling of the bosonic modified-measure string through the auxiliary gauge field A_a to a conserved world-sheet current $\varepsilon^{ab}\partial_b v$ where v is a world-sheet scalar field:

$$S=-\int d^2\sigma\,\Phi(\varphi)\left[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu + \frac{1}{2}\gamma^{ab}\partial_a v\,\partial_b v - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}}F_{ab}(A)\right] + \epsilon\int d^2\sigma A_a \varepsilon^{ab}\partial_b v. \tag{A5}$$

Notice that the last term in Eq. (A5) can be rewritten in the form

$$\epsilon\int \sqrt{-\gamma}d^2\sigma\,A_a \frac{\varepsilon^{ab}}{\sqrt{-\gamma}}\partial_b v, \tag{A6}$$

which means that by including this term we study a model which belongs to the class of *two measure theories* [5].[5]

The equations of motion with respect to A_a read

$$\varepsilon^{ab}\partial_b\left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}+\epsilon v\right)=0, \quad \text{i.e.,} \frac{\Phi(\varphi)}{\sqrt{-\gamma}}=C-\epsilon v, \tag{A7}$$

where C is a dynamically generated constant scale. The canonical Hamiltonian treatment of Eq. (A5) is completely analogous to the simpler case of Eq. (16) in Sec. II. In particular, for the auxiliary "electric" field strength we obtain

$$\pi_{A_1}\equiv E=\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\to E+\epsilon v=C \tag{A8}$$

[cf. Eq. (A7)] and the canonical Hamiltonian becomes

$$\mathcal{H}=-\frac{1}{\sqrt{-\gamma}\gamma^{00}}\frac{1}{2}\left[\frac{1}{E}\mathcal{P}^2+E(\partial_\sigma X)^2 + \frac{1}{E}(\pi_v+\epsilon A_1)^2+E(\partial_\sigma v)^2\right] + \frac{\gamma^{01}}{\gamma^{00}}[\mathcal{P}_\mu\partial_\sigma X^\mu+(\pi_v+\epsilon A_1)\partial_\sigma v]. \tag{A9}$$

We have skipped in Eq. (A9) the linear combination of the rest of the primary (28) and secondary (32) constraints which remain unaltered by the presence of the new field v except for the "Gauss-law" constraint which now reads [cf. Eq. (A8)]

$$\partial_\sigma(E+\epsilon v)=0. \tag{A10}$$

One can check that the basic constraints entering Eq. (A9) span again a closed Poisson-bracket algebra which this time involves also the "Gauss-law" constraint (A10) and the following *variable* string tension equal to the world-sheet "electric" field (A8):

$$T\equiv E=C-\epsilon v. \tag{A11}$$

[5] In D-dimensional space-time the action generically has the form $S=\int d^D x\,\Phi(\varphi)L_1+\int d^D x\,\sqrt{-\gamma}L_2$, where the Lagrangian densities L_1 and L_2 are independent of the degrees of freedom φ^i building up Φ.

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